Nuclear Theory - Course 227

THERMAL REACTORS (BASIC DESIGN)

When a U-235 nucleus fissions an average of 2.5 neutrons are released in addition to the energy. This suggests that these neutrons could be used to cause additional fissions thus creating a chain reaction. Assume we have natural uranium (0.72% U-235) as fuel. Let us start with some numbers of fission neutrons (say 1000). These neutrons have an average energy of about 2 MeV (see Table 4, lesson 227.00-2). Since the fission cross-section of U-235 is about 1 barn at this energy it seems reasonable to "slow" the neutrons to low energy where the fission cross-section is much higher (580 barns for a neutron energy of 0.025 eV). We can rapidly reduce the energy of a neutron by having it undergo elastic collisions with a light nucleus (H, He, C, etc). As a result we obtain neutrons which are in thermal equilibrium with their surroundings and are therefore called "thermal neutrons". At a temperature of 20°C a thermal neutron has an energy of 0.025 eV.

During the slowing down process the neutrons pass through the resonance absorption energies of the U-238. If the fuel and moderator were intimately mixed (homogeneously) too many of the neutrons would suffer resonance capture and a chain reaction could not be sustained. If, however, we separate the fuel into discrete locations within the moderator, ie, using fuel channels, the neutrons can slow down away from the U-238 thus avoiding resonance capture.

At thermal energies the neutrons diffuse around until they are absorbed by the fuel, or leak out of the reactor, or are absorbed by something other than the fuel (moderator, fuel sheath, pressure tubes, etc). By careful choice of reactor materials we can limit the non-fuel or parasitic absorption. In addition we can reduce leakage by careful design of the size and shape of the reactor. Of the neutrons absorbed by the fuel, some will cause fission while others will simply undergo radiative capture. If enough of the neutrons cause fissions to give us the 1000 neutrons we started out with then we have a self-sustained chain reaction. We can define a neutron multiplication factor (k) for this reaction as:

$$k = \frac{\text{number of neutrons in one generation}}{\text{number of neutrons in the preceding generation}} \, *$$

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*Note that this definition is only valid when the effects of source neutrons (photoneutrons and spontaneous fission neutrons) are negligible.

For the chain reaction to be self-sustaining $k \geq 1$. We will deal with k in more detail in the next lesson. The remainder of this lesson will deal with moderator properties, reducing neutron leakage, and spacing of the fuel channels.

Moderator Properties

The primary objective of a moderator is a lot easier to express than to achieve: the fission neutrons must be slowed down to thermal energies without being absorbed. Let us examine the latter aspect first:

There are two possibilities:- the neutrons can be absorbed by the moderator atoms themselves or by fuel atoms, and this can occur anywhere in the energy range from $\sim$2 MeV (fission neutrons) down to 0.025 eV (thermal neutrons). Absorption by moderator atoms can obviously be minimized by choosing a moderator with a sufficiently low absorption cross-section, but for fuel the argument is rather more subtle.

Recall that U-238 exhibits a number of severe absorption peaks between 5 eV and 1 keV. It is essential to minimize resonance capture, and one way of doing this is to ensure that, in the slowing down process, the neutron energy loss per collision is as high as possible. For example, consider the moderators in Figure 1 (for the sake of simplicity the resonances have been smoothed out).

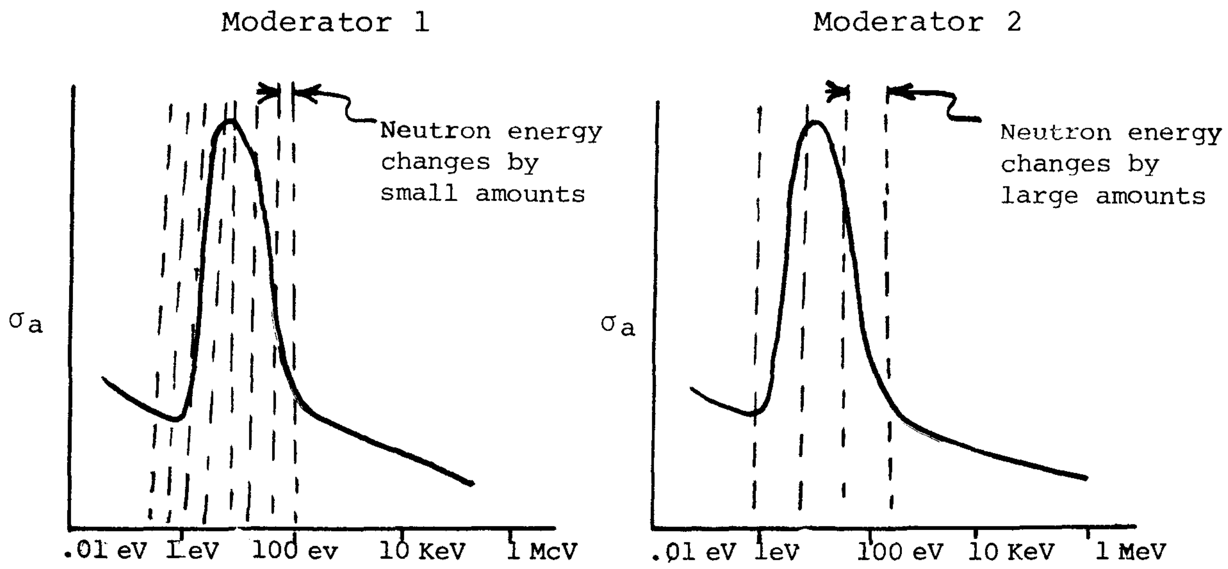

Figure 1

Effect of Moderator on Resonance Capture

Moderator 2 thermalizes the neutrons in far fewer collisions that Moderator 1. This means that the neutrons in Moderator 2 will spend less time in the resonance energy region, and will therefore also have less chance of colliding with U-238 while they have this energy. If they do collide, they will almost certainly be captured. The conclusion is that there will be less resonance capture in U-238 with Moderator 2 than with Moderator 1.

Slowing Down Mechanism

Having established that we want to slow the neutrons down in as few collisions as possible we shall now examine how this might be achieved.

There are two slowing down mechanisms:

(1) inelastic scattering (with fuel nuclei)
(2) elastic scattering (with moderator nuclei)

(Inelastic scattering with moderator nuclei is not possible because the neutron energies are too low, and even with uranium nuclei it is only possible down to about 100 keV. In any case, it is relatively unimportant. Elastic scattering with fuel nuclei may be ignored, because the energy loss per collision is negligible).

In an elastic collision the energy lost by a neutron depends on the mass of the target nucleus and the angle of collision. Since the angle of collision is totally random, a mathematical function can account for its effect. By manipulating equations for conservation of momentum and conservation of kinetic energy we could prove that, the most energy is lost when a neutron collides with a target of equal mass and that for targets of general mass A the energy lost is a simple function of the mass of the target.

It takes a number of collisions for a fast (2 MeV)neutron to slow to thermal energy (0.025 eV) and the larger the mass of the target nucleus, the larger the number of collisions required. This is due to the fact that a smaller portion of the neutron's energy is lost per collision. The mathematical function used to express this is the mean logarithmic energy decrement ξ (xi).

$$N\xi = \text{Ln}\ \frac{E_i}{E_f}$$

Where:
ξ = mean log energy decrement
N = number of collisions to thermalize
E_i = initial neutron energy
E_f = final neutron energy

and $N\xi$ = total E loss going from E_i to E_f

Table I shows the accurate values of ξ of a number of light materials which might be suitable as moderators.

TABLE I

Mean Logarithmic Decrements

	ξ	Collisions to Thermalize
H^1 *	1.000	18
H^2 *	0.725	25
He^4*	0.425	43
Be^9	0.206	83
C^{12}	0.158	115
H_2O	0.927	20
D_2O	0.510	36
BeO	0.174	105

*Gases at STP

Slowing Down Power and Moderating Ratios

A small number of collisions to thermalize is obviously desirable, but this is of no use on its own unless the collisions actually occur. This implies that the probability of a collision must be high, that is Σ_s should be large. Recall that:

$$\Sigma_s = \sigma_s N$$

This immediately rules out gases as moderators, because N would be too small for the neutrons to be slowed down within a reasonable distance.

The overall effectiveness of a material for slowing down neutrons is measured by the product $\xi\Sigma_s$ which is known as the *Slowing Down Power*.

Table II shows the slowing down powers of the solid and liquid moderators. The value of the slowing down power is also shown to demonstrate the unsuitability of a gas.

TABLE II

Slowing Down Powers and Moderating Ratios

	ξ	$\Sigma_s\,(cm^{-1})$ [a]	$\xi\Sigma_s$	Σ_a	$\xi\Sigma_s/\Sigma_a$
He [b]	0.425	21×10^{-6}	9×10^{-6}	? very small	? large
Be	0.206	0.74	0.15	1.17×10^{-3}	130
C [c]	0.158	0.38	0.06	0.38×10^{-3}	160
BeO	0.174	0.69	0.12	0.68×10^{-3}	180
H_2O	0.927	1.47	1.36	22×10^{-3}	60
D_2O	0.510	0.35	0.18	0.33×10^{-4} [d]	5500 [d]
D_2O	0.510	0.35	0.18	0.88×10^{-4} [e]	2047 [e]
D_2O	0.510	0.35	0.18	2.53×10^{-4} [f]	712 [f]

(a) Σ_s values of epithermal neutrons (ie, between $\sim$1 and $\sim$1000 eV)
(b) at S.T.P.
(c) reactor-grade graphite
(d) 100% pure D_2O
(e) 99.75% D_2O
(f) 99.0% D_2O

Not only must the moderator be effective in slowing down neutrons, but it must also have a small capture cross-section. Neutrons are slowed down to decrease radiative captures compared to fission captures, and obviously the whole purpose of moderation would be defeated if the moderator nuclei themselves captured neutrons.

A reasonable indication of the overall quality of a moderator is the *Moderating Ratio*, which combines the slowing down power and the macroscopic capture cross section:

$$\text{Moderating Ratio} = \frac{\xi\Sigma_s}{\Sigma_a}$$

We are now in a position to draw some interesting conclusions from Table I.

H_2O has excellent slowing down properties, and is often used as a fast neutron shield (neutrons must be slowed down before they can be absorbed. Why?) Unfortunately its Σ_a is too high to permit its use as a moderator for natural uranium fuel, and enrichment is necessary.

Be, BeO and graphite have lower values of Σ_a, and can be used with natural uranium fuel provided the fuel is in metal form. The use of natural uranium compounds with more attractive physical and chemical properties (such as UO_2 or UC) is not feasible with these moderators, because of the reduction in the concentration of uranium atoms. The British line of power reactors used a graphite moderator with natural uranium metal fuel (their earlier Magnox stations), and in the early '60s they changed to graphite with enriched UO_2 fuel (the AGR stations).

In the U.S., an abundance of U-235 produced for weapons and a tradition of using it in nuclear submarines led to all out development of light water reactors with relatively highly enriched fuel and a relatively poor moderator.

You can see from Table I that heavy water is by far the best moderator as far as its nuclear properties go, and of course its use was adopted for the CANDU line of reactors (CANDU = CANadian-Deuterium-Uranium). Its Σ_a is so low that natural uranium can even be used in compound form as UO_2.

The substance used as a moderator must be very pure. It is usually used, in a reactor, in larger amounts than any other material, eg, the volume of carbon in a graphite moderated reactor is 70 to 80 times that of the fuel. A very small amount of impurity in a moderator can substantially increase its capture cross-section. The addition of 1 boron atom to every million graphite atoms would increase the capture cross-section of graphite by 25%

For the same reason the isotopic purity of D_2O must be kept high. The addition of 0.25% H_2O to pure D_2O more than doubles the capture cross-section. Thus, the isotopic purity of moderator D_2O is kept at 99.75% by weight or better. This is known as *reactor-grade* D_2O. As you might surmize from the moderating ratio, it would be difficult if not impossible to keep the reactor critical if the isotopic were allowed to drop to 99%. For a more practical approach, Figure 2 shows the change in reactivity with moderator isotopic. Downgrading of the moderator by <u>only</u> 0.1% will introduce about -4 mk. (By contrast downgrading of the heat transport fluid is less important simply because the volume is much less. Thus, downgrading of the heat transport fluid to 95% isotopic will introduce about -5 mk.)

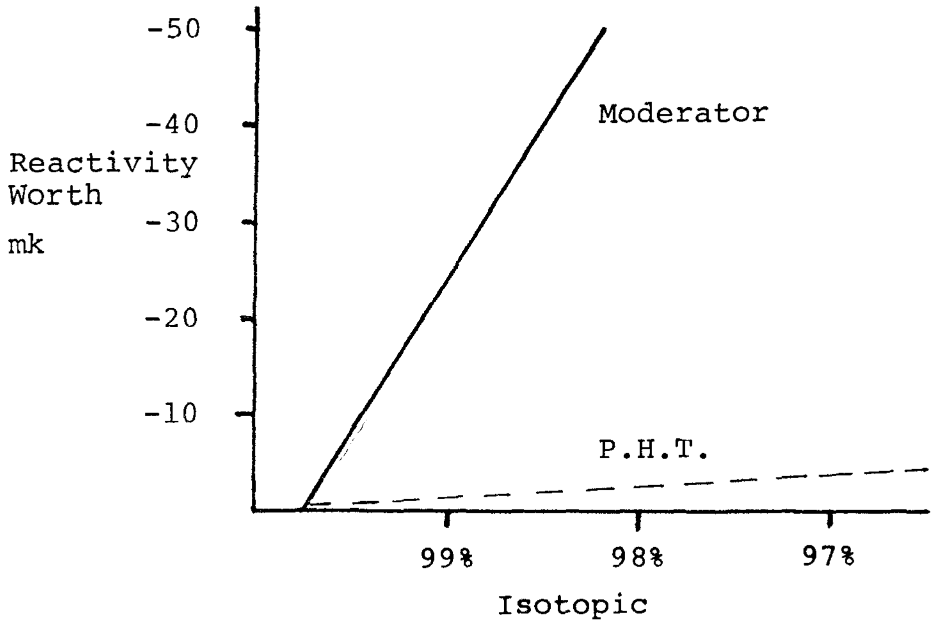

Figure 2

Effect of Moderator or P.H.T. Downgrading

The Diffusion of Neutrons Through the Moderator

Many parameters which determine the design of a reactor are dependent on the way neutrons are slowed down and diffuse in the moderator.

Neutrons diffuse through a material as a result of being scattered by nuclei. Neutrons virtually never collide with each other because the neutron density is so much smaller than the atomic density. The treatment of neutron diffusion, which is a process similar to the diffusion of electrons in a metal, is too complicated to include in this course, and we shall therefore restrict ourselves to the pictorial representation in Figure 3.

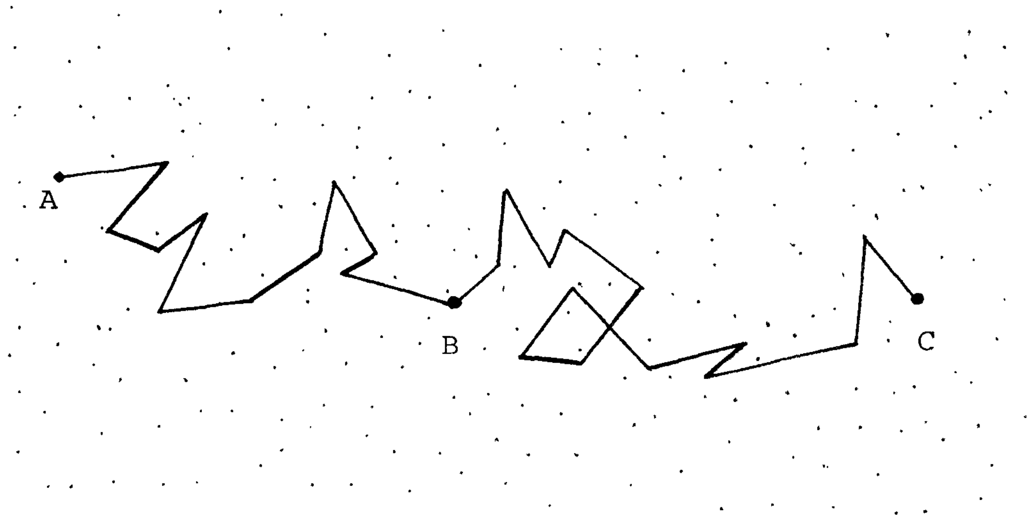

Figure 3

Neutron Diffusion In A Moderator

A fission neutron born at A is thermalized after several collisions and arrives at B. The average distance between A and B in a Candu reactor is about 25 cm. After slowing down the neutron diffuses to C where it is absorbed. The distance BC is about 30 cm. These are known as the mean "crow-flight" distances and are straight line displacements not total distance travelled.

The crow-flight slowing down distance determines the optimum distance between adjacent fuel channels. This spacing is called the *Lattice Pitch*. Figure 4 shows the approximate variation of k (the neutron multiplication factor), with lattice pitch.

Note that if the lattice pitch varies in either direction from its optimum value, k will decrease. If the pitch is increased we have extra moderator and some neutrons are being unnecessarily absorbed by the moderator before they reach the fuel. In this case the reactor is said to be overmoderated. If the pitch is decreased, we don't have enough moderator and some neutrons reach the fuel while still at resonance energy thus more are absorbed wastefully by the U-238 resonances. In this case the reactor is undermoderated.

All of Ontario Hydro's reactors are overmoderated. The reasons for this are physical rather than nuclear. The pressure tubes must have sufficient separation to allow the

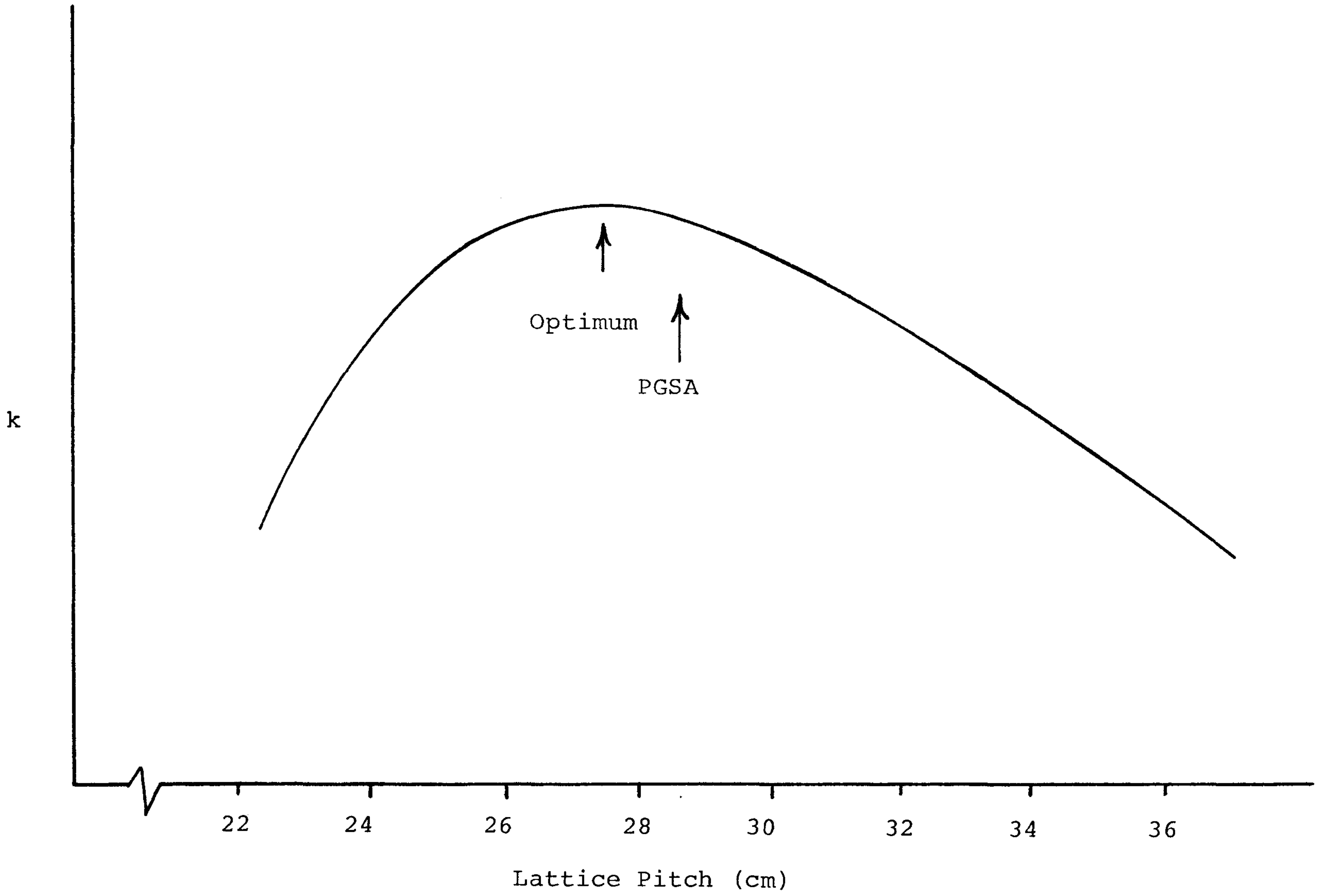

Figure 4

k vs Lattice Pitch

fueling machine access to the end fitting on either end of the fuel channel and the calandria tubes must have sufficient separation to allow space horizontally and vertically for control mechanism guide tubes.

Overmoderating our reactors by a small amount has very little effect as you can see by looking at the decrease in k on Figure 4 for the PGSA reactors.

The behaviour of k with Lattice Pitch has an additional benefit in that any significant accidental rearrangement of the reactor structure makes the reactor less reactive, for example, core disassembly in a melt down.

ASSIGNMENT

1. What is the practical significance of the fact that D_2O is poorer at slowing down neutrons than H_2O?
2. Explain why changing the lattice pitch from the optimum value causes a decrease in reactivity. Why is this a safety feature?

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